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## The Golden Ratio

The Golden Ratio is sometimes called the *Golden Section* or the *Divine Proportion*. Three points A, B and C divide a line in this proportion if  $\frac{AC}{AB} = \frac{AB}{BC}$



This ratio is sometimes called  $\phi$  and has a value of about 1.618. If we let  $BC = 1$  it follows that  $AB = \phi$  and  $AC = \phi^2$ . Since  $AC = AB + BC$  we deduce that  $\phi^2 = \phi + 1$ .

By solving the quadratic we find  $\phi = \frac{\sqrt{5} + 1}{2}$

Artists and architects have long held that rectangles with sides in this ratio offer the most pleasing aspect ratio. If only the designers of 16:9 TV screens knew this.

The Golden Ratio is constantly at hand. You may have watched a documentary on SBS which describes the layout of buildings on the Acropolis. The Golden Ratio was used extensively to derive the proportions of the Parthenon and the detail of the Erechtheion.

Many of you, particularly subscribers to the *AAMT-List*, will have recently bought a copy of *Donald in Mathmagicland* and found that it includes a section about the Golden Ratio and the ratios within a five pointed star or pentagram. You may have relaxed during your vacation reading *The Da Vinci Code* where you would have read a more intriguing interpretation of these matters.

Here are two computing exercises which provide follow up activities after your class watches *Donald in Mathmagicland*.

The Golden Ratio is closely linked with the Fibonacci sequence {1,1,2,3,5,...} for which new terms are generated by adding the two previous terms. If we count the clockwise and counter-clockwise spirals formed by the seeds of sunflowers or pinecones the numbers are consecutive Fibonacci numbers and new seeds are offset at an angle of  $2\pi/\phi$ . The convergence of the ratios of consecutive Fibonacci numbers to the Golden Ratio suggests this simple spreadsheet exercise.

This spreadsheet shows the Fibonacci sequence in Column A [copy down  $A3 = A1 + A2$ ]. The quotients of consecutive terms converge to  $\phi = 1.618\dots$  in Column B [copy down  $B1 = A2/A1$ ].

	A	B
1	1	1.0000000000000000
2	1	2.0000000000000000
3	2	1.5000000000000000
4	3	1.6666666666666667
5	5	1.6000000000000000
6	8	1.6250000000000000
7	13	1.61538461538462
8	21	1.61904761904762
9	34	1.6176470582353
10	55	1.61818181818182
11	89	1.61797752808989
12	144	1.6180555555556
13	233	1.61802575107296
14	377	1.61803713527851
15	610	1.61803278688525
16	987	1.61803444782168
17	1597	1.61803381340013
18	2584	1.61803405572755
19	4181	1.61803396316671
20	6765	1.6180339852180
21	10946	1.61803398501736
22	17711	1.61803399017560
23	28657	1.61803398820532
24	46368	1.61803398895790
25	75025	1.61803398867044
26	121393	1.61803398878024
27	196418	1.61803398873830
28	317811	1.61803398875432
29	514229	1.61803398874820
30	832040	1.61803398875054
31	1346269	1.61803398874965
32	2178309	1.61803398874999
33	3524578	1.61803398874986
34	5702887	1.61803398874991
35	9227465	1.61803398874989
36	14930352	1.61803398874990
37	24157817	1.61803398874989

This simple *JavaSketchpad* exercise explores the Golden Ratios within a five pointed star. The advantage of *JavaSketchpad* is that students can use it on their home computers free of charge. They need to download the jsp4.JAR applet from [www.keypress.com/sketchpad](http://www.keypress.com/sketchpad) and success will depend on the version of Java used in their machine's operating system. It all works fine on recent machines with new browser software.

Students should open to the source code of a blank browser screen and type in the code shown below. The file should then be saved as an html file in the same folder as the jsp4.JAR applet. On opening the html file the five pointed star should appear. The active point is J and wherever J is moved relative to the centre C, the ratios of the segments along the line JL remain constant with a value of  $\phi = 1.618\dots$

```
<HTML>
<HEAD>
<TITLE>Pentagram</TITLE>
</HEAD>
<BODY>
<APPLET codeBase=jsp height=310 width=250
archive=jsp4.JAR align=left code=GSP.class>
<PARAM NAME="Frame" VALUE="1">
<PARAM NAME="MeasureInDegrees" VALUE="1">
<PARAM NAME=BackRed Value=255>
<PARAM NAME=BackGreen Value=255>
<PARAM NAME=BackBlue Value=255>
<PARAM NAME="DirectedAngles" VALUE="0">
<PARAM NAME="Construction" VALUE=""
{1} Point (120,120) [label('C')];
{2} Point (120,220) [label('J')];
{3} Circle (1,2);
{4} Rotation (2,1,1.256637) [label('K')];
{5} Rotation (4,1,1.256637) [label('L')];
{6} Rotation (5,1,1.256637) [label('M')];
{7} Rotation (6,1,1.256637) [label('N')];
{8} Segment (2,5);
{9} Segment (2,6);
{10} Segment (4,6);
{11} Segment (4,7);
{12} Segment (5,7);
{13} Intersect (10,12) [label('P')];
{14} Intersect (9,12) [label('Q')];
{15} Intersect (9,11) [label('R')];
{16} Intersect (8,11) [label('S')];
{17} Intersect (8,10) [label('T')];
{18} Segment (2,17);
{19} Segment (17,5);
{20} Segment (16,17);
{21} Ratio/Segments ( 8,18,50,250,'Ratio JL/JT = ');
{22} Ratio/Segments (18,19,50,270,'Ratio JT/JS = ');
{23} Ratio/Segments (19,20,50,290,'Ratio JS/ST = ');
">
</APPLET>
</BODY></HTML>
```

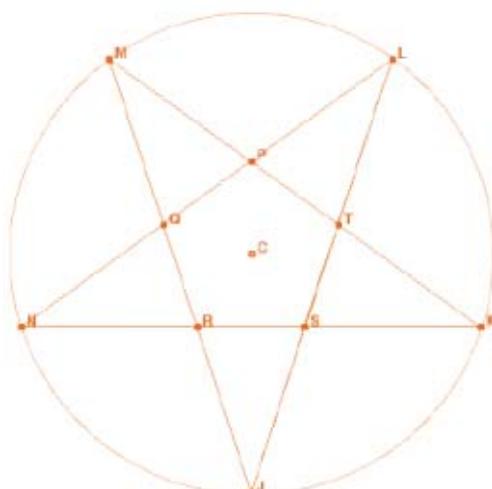
The <APPLET> line allows us to adjust the dimensions of the frame to fit a particular screen. Construction items are numbered {1}, {2} etc. The first item, {1}, is the point C at the centre of the circle. Point J, item {2}, is then defined below C.

Item {3} is coded circle (1,2) which means its centre is at C, {1}, and it passes through J, {2}.

Item {4} defines a new point K which is obtained by rotating J anticlockwise about C through the angle  $72^\circ$ . But, even though MeasureInDegrees is switched on with VALUE="1", we have to express the angle  $72^\circ$  in radian measure ( $2\pi/5 \approx 1.256637$ ).

Similarly we define L, M and N with further rotations by  $72^\circ$ . From {8} to {12} we draw the segments joining J, K, L, M and N and from {13} to {17} their intersection points at P, Q, R, S and T. Then we join the points JL, JT and JS so that each of these segments has an item number. Finally we evaluate the ratios of these segments and find that they are each equal to the Golden Ratio. You could use *Cabri* or *GSP* just as easily. Although the Java Applet can be used within the same html code as JavaScript, I do not know of any way to pass JavaScript values to the Java applet.

*JavaSketchpad* provides a simple way to describe such constructions in this format and it has the added advantages of being free and easily shared using the Internet. Using *JSP* our students are much more likely to explore these ideas at home and catch something of Da Vinci's and Disney's fascination with these topics.



$$\begin{aligned}\text{Ratio JL/JT} &= 1.618 \\ \text{Ratio JT/JS} &= 1.618 \\ \text{Ratio JS/ST} &= 1.618\end{aligned}$$

